

Full Length Research Paper

The effect of geometry instruction with dynamic geometry software; GeoGebra on Van Hiele geometry understanding levels of students

Tamer Kutluca

Dicle University, College of Education, Diyarbakır, Turkey.

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The aim of this study is to investigate the effect of dynamic geometry software GeoGebra on Van Hiele geometry understanding level of students at 11th grade geometry course. The study was conducted with pre and posttest control group quasi-experimental method. The sample of the study was 42 eleventh grade students studying in the spring term of 2011 to 2012 educational year. "Van Hiele Level of Geometric Understanding Test" developed by Usiskin (1982) and translated into Turkish by Duatepe (2001) was used as the data collecting tool of this study. Van Hiele Level of Geometric Understanding Test has 25 items. This result can be explained by the computer assisted instructional method applied in the experimental group and the learning environment provided by the dynamic geometry software GeoGebra. As a matter of fact, unlike the students in the control group, the students in the experimental group had the opportunity of moving given shapes or creating their own geometric shapes, trying different things on the shapes, testing and constructing their own knowledge. In addition, thanks to the activities in the learning environment, the students in the experimental group also had opportunity to participate actively to the instructional process, to share ideas comfortably, to discuss about the obtained results with friends and to construct their own knowledge.

Key words: Trigonometry, dynamic mathematics software, GeoGebra, achievement, attitude.

INTRODUCTION

Geometry is an important study field of mathematics. Geometry not only does help students to associate geometric patterns in the universe with several branches of mathematics, but also enables students to apply knowledge they learned through geometry topics on problem solving, daily life and other courses. Apart from the mentioned benefits, learning geometry contributes to improve basic skills of students like analyzing, comparing and generalizing. It also equips students with scientific thinking skills like examining, researching, being critical, schematizing thoughts, being tidy, careful and patient, and expressing thoughts clearly (Baki, 2002; Kılıç, 2003). One of the studies about development of geometric

thought of individuals was conducted by two Dutch educationalists Pierre Van Hiele and Dina Van Hiele Geldof. They developed a model describing geometrical understanding level of children by focusing on problems students face when they learn geometry (Olkun and Toluk, 2003). According to the Van Hiele's (1986) theory the geometric understanding level of children follows five stages. They are "Visual Level", "Descriptive/Analytic Level", "Abstract/Relational Level", "Formal Deduction Level", and "Mathematical Rigor Level" respectively.

Although geometry provides numerous benefits, primary and secondary school students in Turkey scare and do not like geometry related topics and eventually

fail. As a matter of fact Turkey was 34th out of 38 countries in the 3rd Trends in International Mathematics and Science Study (TIMSS) in 1999 (Monet, 2003). Based on TIMSS 1999 results Turkey was also well below the international average in Geometry subtitle. Olkun and

Aydoğdu (2003) reported one of the reasons of failure in geometry as teachers' directing students towards memorizing during the period they acquire geometric knowledge and skills. Toluk (2005) indicated condensed existence of the geometry topics in the curricula as another reason for the failure. The problems faced in geometry instruction in our country revealed that various instructional materials should be developed and applied in geometry instruction.

Results of a great deal of studies have shown that Computer Assisted Instruction (CAI) has significant effect on Van Hiele geometry understanding level of students. Some of these studies are listed as follows. Breen (2000) determined computer assisted geometry instruction at 8th grade positively affected Van Hiele geometry understanding level and understanding geometry concepts. Similarly in the experimental study, Assaf (1986) determined that using Logo in geometry instruction significantly increased geometric understanding, geometry knowledge and attitudes towards geometry levels of students comparing to the traditional method. Scally (1991) found out that experiences in Logo learning program increased geometry understanding level of students. In the study with 6th to 8th graders (second stage of elementary), Frerking (1994) came up with that using "Geometry Sketchpad" software in geometry instruction significantly enhanced Van Hiele geometry understanding level and proving skills of students comparing to traditional method.

Reviewing the studies above, it is seen that the previous studies on CAI on Van Hiele geometry understanding levels of students have generally been conducted on elementary school level. Therefore there is need for studies examining the same topic at secondary school level. The recommended units by Higher Education Council for the geometry course are axiom, theorem and their relations, angles, triangular, tetragons, polygons, circle, circular area and angle, length and area concepts of them, and finally area and volume of solid shapes. Many studies have been working for nearly fifteen years with several mathematical packages is used are seen in the literature such as; *Logo, Geometer's Sketchpad and GeoGebra* (Reis, 2010; De Villiers, 2004; Johnson, 2002; Frerking, 1994; Assaf, 1986). The software that provide visual and effective learning environment for students have increased as Technological improvements have increase. One of these software, GeoGebra, can be defined as Computer Algebra System (CAS) since it includes the symbolical and visualization features such as direct coding of equations and coordinates, defining functions as algebraically. It can also be defined as Dynamic Geometry Software

(DGS) since it includes concepts such as points, segments, lines, conic segments and provides dynamic relationships between the concepts. It is the basic feature of GeoGebra that can be approached both CAS and DGS. In education of geometry, the ability of software making a relationship between geometry and algebra has become an important value in math curriculum (Hohenwarter and Jones, 2007).

During the past decades, there has been a great evolution in mathematical software packages. Of a great amount of software, there are two important forms of software contributing to the teaching and learning of mathematics; CAS and DGS. These two tools have had a high influence on mathematics education. However, these are not connected to each other at all. Fortunately there is a software system called GeoGebra that integrates possibilities of both dynamic geometry and computer algebra in one program for mathematics teaching (Hohenwarter and Jones, 2007; Dikovic, 2009).

Within this framework a CAI material was developed and applied with GeoGebra, dynamic geometry software, for the instruction of "circle" unit in the 11th grade geometry course. GeoGebra software, which is a versatile and practical tool, can be used variety of ways in geometry. The School Curricula prioritized the skill of associating geometry and algebra (Hohenwarter and Jones, 2007). For this reason GeoGebra software, which enables students examine circles both in algebraic and arithmetic way, was used in this study.

Purpose of the study

The aim of this study is to investigate the effect of dynamic geometry software GeoGebra (DGSG) on Van Hiele geometry understanding level of students at 11th grade geometry course. For this aim, the following research questions were pursued.

1. Is there significant difference between pretest scores of the experimental and control group students?
2. Is there significant difference between pretest and posttest scores of the experimental and control group students?
3. Is there significant difference between posttest scores of the experimental and control group students?

METHOD

In the study, pre-tests and post-tests were given to the participants before and after the instruction as an independent variable. The study was conducted with pre and posttest control group quasi-experimental method. Because "it provides the best approach to investigating cause and effect relationships" (McMillan, 2000, p. 207). During the study the experimental group received computer-assisted instruction with the dynamic geometry software GeoGebra as a supplementary teaching tool, and the control group received instruction by means of traditional teaching. Accordingly, the

students in the experimental and control groups were not assigned randomly but the equivalency of the groups was emphasized instead. For this reason, the researcher has to designate in an unbiased manner one group as the experimental group and the other as the control group. In such situations, the quasi-experimental design is used (Melanlioğlu, 2013). The quasi-experimental method involves an experimental approach in which random distribution is not used while appointing participants to experimental and control groups (Creswell and Clark, 2007).

This school was chosen deliberately because it was the state primary school with a computer laboratory that was located in a middleclass socio-economic neighborhood.

Working group

The sample of the study was 42 eleventh grade students studying in the spring term of 2011 to 2012 educational year.

There were 42 students, 24 students from class in the experimental group which received computer assisted instruction (CAI) with dynamic geometry software GeoGebra during laboratory sessions. The students in the control group consisted of 18 students, who experienced traditional instructional methods (such as questioning, solving exercises, etc.).

Although there is no computer literacy course in the National Curriculum, the school administration provides computer literacy courses once a week for eleventh grade level. Therefore, all the participants of the study were capable of using computers effectively.

Experimental process

Before starting experimental phase while experimental and control groups were being determined the opinions of the course teacher were collected and 11-A and 11-B classrooms with equivalent achievement level were included. Van Hiele Geometry Level Understanding Test, developed by Usiskin (1982) and translated into Turkish by Duatepe (2001), was applied on the experimental and control groups. As a result of the pretest applied on the experimental and control group no significant understanding level differences was detected between the two groups. This situation indicated that the experimental group and the control group were equivalent in terms of geometric understanding level. At the end of the experimental process "Van Hiele Geometry Level Understanding Test" was applied to experimental and control groups as the posttest.

Instruction in experimental group

In the experimental group the instruction of the "Circle" unit from 11th grade geometry curriculum was carried out with computer assisted instructional activities which use dynamic geometry software GeoGebra, developed by the researcher. The computer assisted instructional activities were performed in the computer laboratory. Minimum groups of 2 students were sharing one computer. The groups were given the worksheets. In the instructional process of the experimental group, the teacher undertook a guide role; he encouraged students to carry on group work and complete activities and let them share knowledge through discussions. The implementation period in the experimental group took three weeks.

Instruction in control group

In the control group the instruction was not manipulated, the flow of

the lesson was fully left to the course teacher instead. The observations and interviews made before the application period revealed that the course teacher generally use lecturing and questioning techniques in his instruction. In addition, in the control group no other activities were applied other than the ones existing in the textbook. During the instruction, examples from the objects used in the classroom and real life were given, rather than using activity or materials.

Developed computer assisted instruction material

Dynamic Geometry Software (DGS) provides roles like investigating constant relations in the structure of an instructional environment, changing variables to fit newly formed situations, making deductions based on experiences, converting provided verbal and visual information into each other, interpreting the shapes, using visuality and making assumptions (Goldenberg, 1999). Most of the relations in geometry course are obtained by means of visual representations of the objects so visual representations are musts for some students to learn geometry. Visual media provided to students not only does contribute to their geometry achievement but also facilitates active involvement of them (Goldenberg, 1999). Thus, DGS GeoGebra was used in the instruction of The Circles, a topic in 11th grade geometry curriculum.

GeoGebra is structurally dynamic. While students are playing with the visual representations of the geometric structures with mouse pointer, they have opportunities to discover constant and abstract mathematical relations in the structures at the same time. Such a learning environment make students discover and construct their own knowledge structure by interacting with computers.

Worksheets were prepared to guide students through application process of computer assisted instructional activities developed by DGS. The worksheets provide the students with clue-type instructions about the activities, instead of giving ready-knowledge directly. In the process of developing DGS mediated computer assisted instructional materials, the opinions of three education field experts and mathematics teachers were collected and the material was finalized with the pilot application in another school.

Structure of computer assisted worksheets

It is known that persistency of the mathematics knowledge of students is deeply relies on their operational and conceptual learning. It is remarked that persistent learning can be managed by adopting contemporary approaches where students are active in learning process and construct their own knowledge instead of traditional approach where teacher directly try to convey information. Considering this framework, Bruner's Discovery Learning and Vygotsky's Social Interaction Learning were referred when the related computer assisted instruction material and worksheets were being developed.

The developed worksheets make students discover knowledge by means of prompting questions instead of direct information transformation because Bruner, as Piaget, argues that students should actively participate to learning process and this active participation can only be managed by discovery learning. The main concern of this method is teachers' guiding students to reach intended knowledge by arranging certain activities instead of "pouring the information" directly. This manner exists in computer assisted worksheet activities where questions in clue characteristic were embedded to direct students towards the intended knowledge.

GeoGebra software was used in computer assisted instructional material developed for the instruction of "circle" unit because the program provides symbolical and visual capabilities like directly pinning equations and coordinates and defining functions in

algebraic way. GeoGebra is quite flexible with enabling users entering data of various characters. This nature of the software helps students discover new information about circle topic.

Subtopic: Circle

Skills: Using technology, mathematical thinking, modeling, reasoning, associating, problem solving, social interaction

Attainments: Explains the circle and angles in the circle. Calculates perimeter of the circle.

Tools: Computer, GeoGebra software.

Material design of circle unit with dynamic mathematics software

Subject: Geometry
Level: 11th
Unit: Circles

Material design

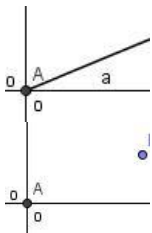
After this step, the following screen is presented as shown in Figure

Material 1



1. Click on 'New Point' icon under 'Point Tool' menu. Form a point by clicking on an empty space on the canvas or clicking on Origin.
2. Click on 'Line Segment with a Given Length from a Point' icon under 'Vector, Ray, Line segment Tools'. Click on the point you previously formed and enter '1' or another number in the dialog box which appears (Figure 1).

Line segment with a given length from a point length OK Cancel



3. Right click on the line segment and choose 'Show Object'

After you choose 'Show Object' the line segment will disappear.

After choosing Show object the object will disappear





4. Right click on point B and left click on  'Show Grid'

Material 2




1. Click on 'New Point' icon under 'Point Tool' tab. Form point A by clicking at the origin on the canvas.
2. Click on 'New Point' icon under 'Point Tool' and form point B (1, 0) 1 unit away from point A.
3. Drop down and enter α as 45° , form a bar on the canvas by clicking on the next sign on the Free Objects tab.
4. Click on 'Angle with Given Degree' icon under 'Number and Angle Tools' tab, enter the value of α into dialog box appearing (Figure 2).
5. Right click on formed point B and choose 'Show Grid' tab (Figure 3)

Material 3

	1.	Form point A (0, 0) at the origin.
	2.	Click on Bar icon under General Tools-1 tab. Form a bar with the name r, minimum value = 1 and maximum value = 6 (Figure 4).
	3.	Click on Bar icon under General Tools-1 tab. Form a second bar with the name n, minimum value = 3 and maximum value = 6 (Figure 4).
	4.	Click on 'Circle with Centre Point and Radius' under 'Circle, Segment and Sector' tab. Enter r in the appearing dialog box or form a circle [A,r] on the canvas.
	5.	Form point B (r, 0)

Material 3. Contd.

	6.	Enter $\alpha = \left(\frac{360}{n}\right)$ form an angle relying on number of sides; n.	
	7.	Degree' icon under 'Number and Angle' tab then choose points A and B and enter α value as	
	8.	icon under 'Polygon Tools' tab. Then choose B and B' respectively. Enter n value into the place appears.	
	9.	like line segment, point, polygon1 in the window on the left when you right click on the object	
	10.	and $t=k/(2r)$ into data entry space.	

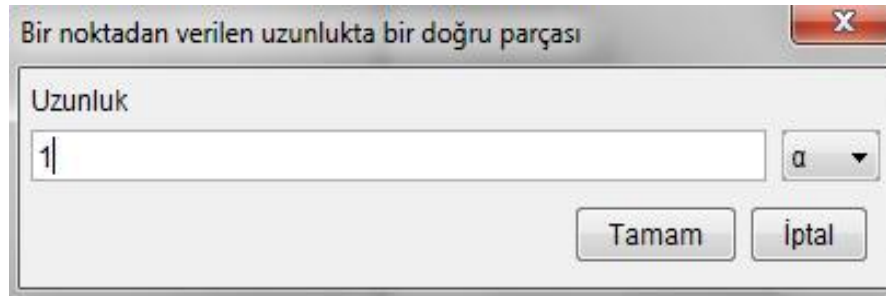


Figure 1. Line segment dialogue box showing a given length from a point length.

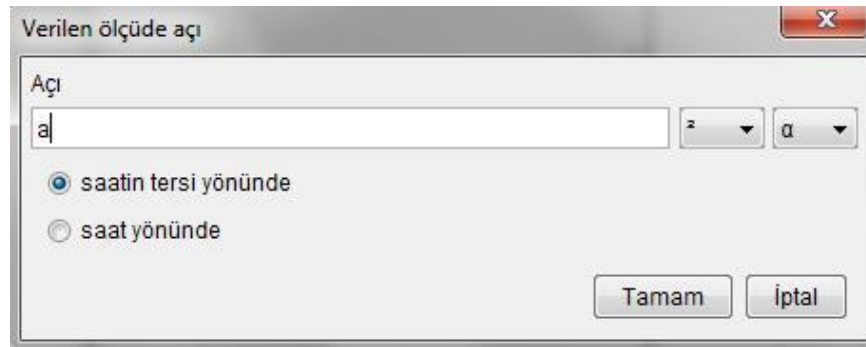


Figure 2. Dialogue box showing 'angle with given degree'.

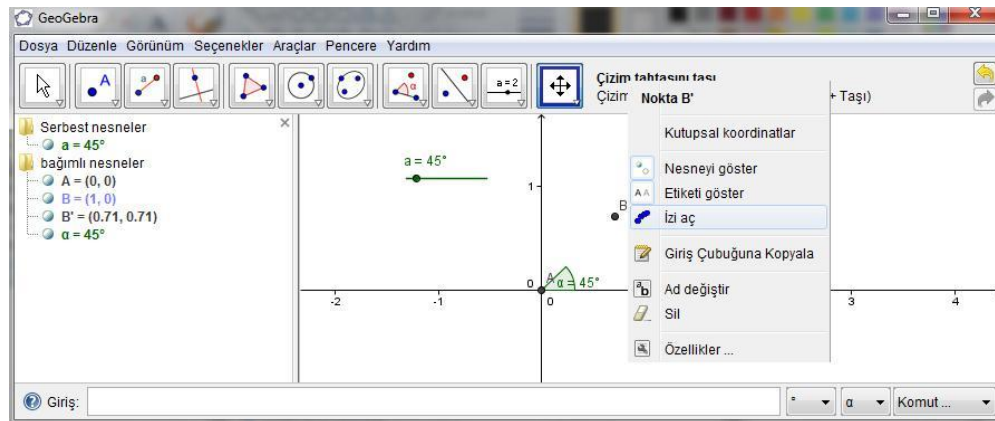


Figure 3. GeoGebra window showing grid tab.

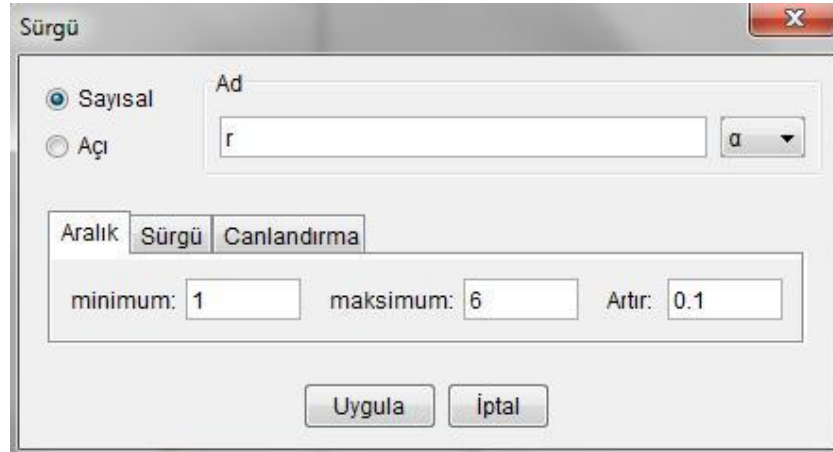


Figure 4. Dialogue box showing the displayed bar icon.

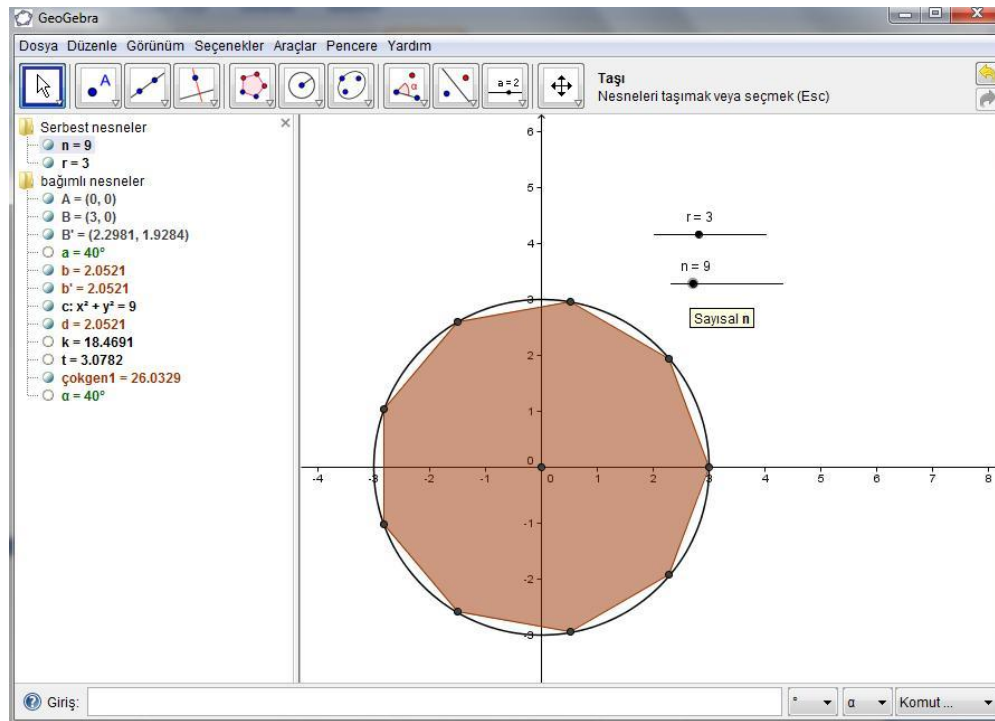


Figure 5. GeoGebra screen showing the obtained Polygon from n and r values.

5.

Teaching and learning process

For material 1

- What kind of route does B follow when you move it?
- Can the route that B follows be expressed with an equation, explain?
- What kind of relation exists between these three shapes?

For material 3

- Move the bar r, explain how the object changes?
- Move the bar n, explain how the object changes?
- How does the perimeter of the polygon change when n changes?
- How does k change when n increases?
- Towards which known number does k approach, explain?
- Can the perimeters of the polygon and the circle be equal? Explain?

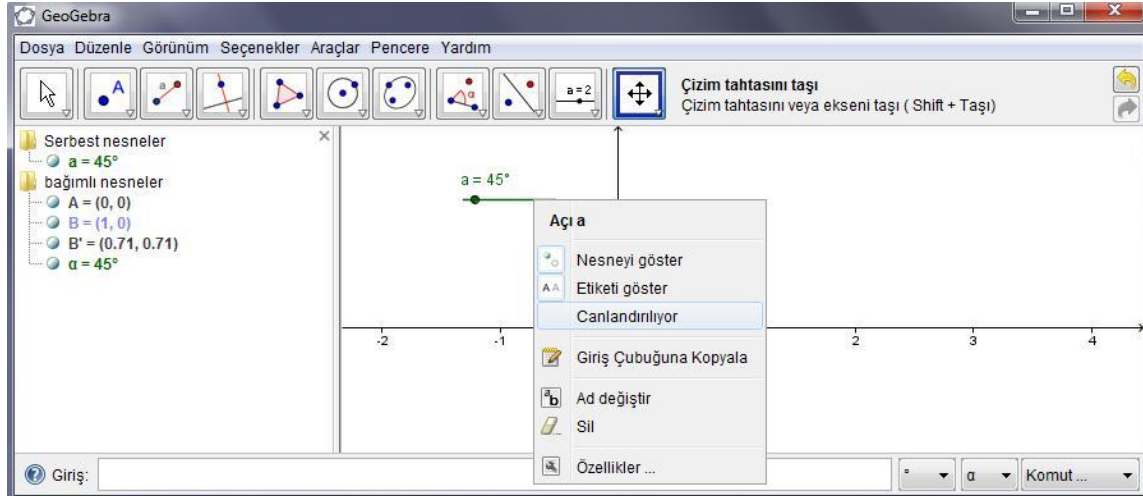



Figure 6. GeoGebra screen showing the axes.

Table 1. Mann Whitney U-test scores for the pretest scores of the groups.

Groups	N	Rank average	Rank total	U	p
Experimental	24	16.46	382.00	128.00	.277
Control	18	20.14	374.00		

Assessment

- Explain: what do you think about the shape formed by right clicking on the bar and choosing Animation tab when the angles are 180° and 360° ?
- After checking the grid as angle, activate Animation tab by right

clicking the bar. Choose three points on the grid  then activate these three points by clicking on Circle Passing through Three Points icon under Circle, Segment Sector Tools tab. Explain the equation appearing on the algebra window (Figure 6).

Data collecting tool

“Van Hiele Level of Geometric Understanding Test” developed by Usiskin (1982) and translated into Turkish by Duatepe (2000) was used as the data collecting tool of this study. Van Hiele Level of Geometric Understanding Test has 25 items. The test has 5 levels and each 5 questions represent a level. Students should answer at least 3 of the 5 questions in each level so that they could be accepted as attain that level. Since the sample of this study were the students of 4th grade in elementary school, the first 15 questions of the test representing the first 3 levels were taken and applied as pre and posttests.

Data analysis

The items of the test were scored as 1 point for right and 0 for the wrong answers. The scoring system by Lee (2000) was used for the scoring of the geometric understanding levels. Therefore if

students:

- Solve the questions and fulfill the criteria of 1st level then they got 1 point,
- Solve the questions and fulfill the criteria of 2nd level then they got 2 points,
- Solve the questions and fulfill the criteria of 3rd level then they got 4 points,
- Solve the questions and fulfill the criteria of 4th level then they got 8 points,
- Solve the questions and fulfill the criteria of 5th level then they got 16 points,

Accordingly a student can get 1 point for the 1st level, 3 for the 2nd, 7 for the 3rd, 15 for the 4th, and 31 for the 5th level maximum. Since the obtained data did not show normal distribution, the pretest scores were analyzed with Mann-Whitney U-Test and Wilcoxon Signed Rank Test applied on SPSS 16.0 statistical pack.

FINDINGS

Is there significant difference between pretest scores of experimental and control groups?

“Van Hiele Level of Geometric Understanding Test” was used as the pretest on the students in the control and experimental groups. Since the distribution of the pretest scores of the groups were not normal, the pretest scores of the groups were compared with Mann-Whitney U test. The test results are shown in Table 1.

Table 2. The results of Wilcoxon signed rank test comparing pretest and posttest scores of the experimental group.

Pretest-posttest	N	Rank average	Rank total	Z	p
Negative rank	1	3.50	3.50	-3.655*	.000
Positive rank	19	9.76	157.50		
Equal	4				

* Based on negative ranks

Table 3. The results of Wilcoxon signed rank test comparing pretest and posttest scores of the control group.

Pretest-posttest	N	Rank average	Rank total	Z	p
Negative rank	3	7.40	22.50	.000*	1.00
Positive rank	7	3.85	22.50		
Equal	8				

*Total of negative ranks and positive ranks are equal.

As seen in Table 1, the rank average of the experimental group is 16.46 and control group is 20.14. This means there is no significant difference between pretest scores of experimental and control groups based on the results of Mann-Whitney U-test [$U=128.00$, $p>.05$]. Therefore it is proved that the experimental and the control groups are equivalent in terms of Van Hiele geometric understanding levels and neither of the groups overruns the other initially.

Is there significant difference between pretest and posttest scores of experimental and control groups?

Wilcoxon signed rank test was applied to test whether there is significant increase in the Van Hiele geometry experimental process applied. The results of Wilcoxon signed rank test comparing pretest and posttest scores of the experimental group are presented in Table 2.

When Table 2 is examined, it can be seen that after the applied experimental procedure there is significant difference between pretest and posttest scores of the students of the experimental group favoring the posttest ($z = -3.655$; $p<0.01$).

This piece of finding obtained shows that there is significant increase in the Van Hiele geometry understanding level of the students in the experimental group as a result of the applied experimental procedure.

Wilcoxon Signed Rank Test results comparing pretest and posttest scores of the control group are presented in Table 3.

When Table 3 is examined, no significant difference exist between pretest and posttest scores of the students of the control group after the lessons with traditional method ($z = .00$; $p>0.05$). This finding proves that there is no significant increase in Van Hiele geometry under-

standing level of the students in control group after the application period.

Is there significant difference between posttest scores of experimental and control groups?

Wilcoxon signed rank test was applied to test whether there is significant increase in the Van Hiele geometry understanding levels of the groups as a result of the experimental process applied. The results of Wilcoxon signed rank test comparing pretest and posttest scores of the experimental group are presented in Table 2.

When Table 2 is examined, it can be seen that after the applied experimental procedure there is significant difference between pretest and posttest scores of the students of the experimental group favoring the posttest ($z = -3.655$; $p<0.01$). This piece of finding obtained shows that there is significant increase in the Van Hiele geometry understanding level of the students in the experimental group as a result of the applied experimental procedure.

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DISCUSSION AND CONCLUSION

Today, there is some of mathematics software available

Table 4. The results of Mann Whitney U-Test comparing posttest scores of experimental and control group.

Groups	N	Rank average	Rank total	U	p
Experimental	24	24.57	565.00	82.00	.004
Control	18	16.54	336.00		

for learning mathematical concepts at the K-12 level. CAI is one application of computers, with different modes, such as drill and practice, tutorials, and simulations. Drill and practice mathematics software presents exercises in an interesting real-life based context in the form of computer games or activities (McCoy, 1996). GeoGebra is dynamic geometry software, as explained in the method section, and is also oriented around general problem solving strategies, interactive exercises about mathematical problems and solutions based on adventure activities.

This study revealed that computer assisted instruction method applied on the experimental group was more effective on increasing Van Hiele geometry thinking levels than traditional method in the instruction of "Circle" unit in the 11th grade geometry curriculum. This result can be explained by the computer assisted instructional method applied in the experimental group and the learning environment provided by the dynamic geometry software GeoGebra. As a matter of fact, unlike the students in the control group, the students in the experimental group had the opportunity of moving given shapes or creating their own geometric shapes, trying different things on the shapes, testing and constructing their own knowledge. In addition, thanks to the activities in the learning environment, the students in the experimental group also had opportunity to participate actively to the instructional process, to share ideas comfortably, to discuss about the obtained results with friends and to construct their own knowledge. This situation caused an increase in the Van Hiele geometry understanding levels of the students in the experimental group. This finding is coherent with the other studies (Erdoğan et al., 2009; Kılıç, 2003).

The result of this study shows that using dynamic geometry software GeoGebra has significant positive effect on Van Hiele geometry understanding level of students coincides with the results of previous studies of Assaf (1986), Scally (1991), Bobango (1988) and Breen (2000), which show that computer assisted instruction in geometry lesson has positive effect on Van Hiele geometry understanding level of students from various levels. However Johnson (2002) came up with no significant difference on academic achievement and Van Hiele understanding level of the students in the study applying dynamic geometry software in geometry lesson. Johnson (2002) explained this result with teachers' converting computer assisted instructional environment into a

traditional one and their insufficient knowledge about DGS.

SUGGESTIONS

In light of the findings of this study, the following recommendations can be suggested for geometry teaching and further research.

The geometry understanding level of students of second stage of elementary or higher level can be improved with the help of computer assisted dynamic geometry software. Geometry courses at primary and secondary schools should be revised besides the instruction at teacher training programs and it should be accomplished that the instruction should be supportive and appropriate to the Van Hiele geometrical thinking levels.

Conveying the opportunities presented by dynamic geometry software into learning environments highly relies on teachers equipped with adequate training. So at pre-service level candidate geometry teachers should be educated and provided with experiences about using dynamic geometry software in geometry instruction and at in-service level teachers should be trained by the experts of this field.

Similar studies should be conducted in different levels of education and with larger study groups.

Geometry topics investigated in this study were circles. The findings of the study cannot be applied to all geometry topics. Furthermore, the study took place over the course of only three weeks. This author thinks that the duration of time given by the schools for the topics to be covered was insufficient. Time constraint also pushed the teachers to limit their instruction and limit the student interaction with each other in the classes.

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